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THERMIC: a 2-D finite-element tool to solve conductive and advective heat transfer problems in Earth Sciences☆

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Abstract

We present a C-language program, THERMIC, that solves the 2-dimensional (pseudo 3D for axi-symmetric cases) conductive and advective heat-transfer equation. THERMIC uses a finite-element method that takes into account realistic geometries, heterogeneous material properties and various boundary and initial conditions. As it also allows for latent heat (heat production due to crystallisation) and for thermal properties, such as thermal conductivity, to be dependent on temperature, it is particularly suited to heat transfer problems encountered in the Earth Sciences. We present sample applications from the various problems already treated by THERMIC (cooling of magma chambers and dykes, the study of a granitic magma ascent or of pore water flow in sedimentary basins).

Successfully tested on SUN® and SGI® UNIX workstations and on Microsoft Windows 95®, 98® and NT® 4.0 system based PCs, the THERMIC package can be downloaded from the web (THERMIC home page: http://www.ipgp.jussieu.fr/UFP/thermic/html/Thermic_home.html) and contains source files, makefiles and environment files as well as executable files for both systems and an html directory with help and example files. © 1999 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Numerous physical problems in Earth Sciences involve heat transport phenomena. Ranging from spatial scales of individual crystals to the entire Earth, conductive and convective heat transfer plays a key role in the behaviour of natural systems. However, analytically exact solutions exist only in a few examples with simple geometry and one must use numerical approaches to obtain quantitatively valid models of these important processes in many geologically realistic situations. A powerful approach to such continuum problems is the finite-element method, which can be viewed as a general discretisation procedure (Zienkiewicz and Taylor, 1989).

THERMIC is a computer program that solves the heat equation using the finite-element method in 2-D space (pseudo 3D in axisymmetric cases). Written in ANSI C, it has been successfully tested on SUN® and SGI® UNIX workstations and on Microsoft Windows 95®, 98® and NT® 4.0 operating system based PCs.

Its main features are:

☆ Code available at http://www.iamg.org/CGEditor/index.htm

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- steady-state or time-varying cases,
- any 2D or axisymmetric geometry using isoparametric elements (3, 4, 6, 8 or 9 nodes),
- fluid circulation (forced convection with a prescribed fluid velocity field),
- handling of various boundary conditions: prescribed heat flow, prescribed temperature, fluid flow and mixed conditions,
- spatially variable material properties,
- temperature-dependent material properties (nonlinear case),
- latent heat of crystallisation (nonlinear case),
- material properties or boundary conditions can be changed at each time step,
- dialogs in French or English and possible extensions to an arbitrary number of languages (a separate ASCII file contains the questions and answers of THERMIC),
- modular ANSI C based conception.

The THERMIC package includes 4 different programs:

- MESHGEN, that generates the finite-element mesh;
- THERMIC, the main heat-equation solver;
- OUT2XYT, that formats THERMIC's output files into x, y, temperature files;
- THERMPRF, that generates temperature vs. time or vs. distance profiles from THERMIC's output files.

Since finite-element theory is now well known (for example Zienkiewicz and Taylor, 1989; Oñate et al., 1991), we will give only a brief review of the theoretical basis. Then, we will present the program itself and some examples of its use.

2. Theory

The general form of the conductive heat equation in a solid having material advection can be written in Cartesian coordinates:

$$\vec{\nabla}(\tilde{K}\vec{\nabla}\Theta) + \rho c\frac{\delta\Theta}{\delta t} + (\rho c)_{\rm f}\tilde{v}_{\rm f}\,\vec{\nabla}\Theta + Q = 0 \quad (1)$$

where Θ is the temperature, $\tilde{K}$ is the thermal conductivity tensor depending on direction and temperature, ρc the heat capacity that can depend on temperature, $(\rho c)_{\rm f}$ the fluid heat capacity and $v_{\rm f}$ its Darcy velocity field and Q the heat production that can also depend on temperature. The boundary conditions for such a solid can be expressed as follows (called mixed boundary conditions hereafter):

$$(\tilde{K}\,\vec{\nabla}\Theta)\,\vec{n} = \Phi_0 - h_{\rm s}(\Theta - \Theta_{\rm a}) \quad (2)$$

where $\vec{n}$ is the unit outward normal vector of the boundary of the model, Φ_0 is the temperature-independent part of the heat flow inward across the boundary and h_s is the coefficient of any temperature-dependent part of the flux inward across the boundary, which is assumed proportional to the difference between Θ and a reference temperature Θ_a.

Using a variational formulation, the heat Eq. (1) can be expressed as a sum of integrals on the surface of a planar solid (2-D case) and on its boundaries. We can then build up a finite-element scheme that consists of dividing the solid into polygonal elements of finite size. Material properties, such as K, ρc and Q are assigned to each element (note that K is the coordinate-aligned orthotropic thermal conductivity, $\tilde{K} = [\begin{smallmatrix} K_x & 0 \\ 0 & K_y \end{smallmatrix}]$). The boundary conditions are applied to the external elements. The temperature is computed at nodes of each element and the solution of the complete system is an assembly of its elements. Thus the solution with THERMIC follows the classical pattern in four steps of any finite-element program:

1. determination of the element properties from the particular geometry and nodal loads;
2. assembly of all equations into a global matrix;
3. introduction of boundary conditions;
4. solution of the resulting linear equation system by triangular decomposition of the global matrix and Gaussian elimination.

Concerning the fluid velocity, one must be aware that it is not the actual velocity of the fluid flowing through a porous medium but the Darcy velocity. In this situation, the fluid keeps its actual density and heat capacity since the Darcy velocity, considered as an average velocity per unit area, already includes the effects of porosity. However, if all of the rock is moving (as in subduction), then the velocity field and the designated fluid density and heat capacity, are those of the moving rock.

A main feature of THERMIC is that it allows a nonlinear solution of the heat equation, with thermal conductivity and heat capacity that can depend on temperature, as well as the influence of the heat of crystallisation.

For the thermal conductivity K, we use the following relationship (Touloukian et al., 1981; Delaney, 1987):

$$K(\Theta) = \alpha_{\rm K} + \beta_{\rm K}/(\Theta + 273.15)$$

where $\alpha_{\rm K}$ and $\beta_{\rm K}$ are coefficients determined empirically (Θ in °C).

For the heat capacity, ρc, we have the following relationship:

$$\rho c(\Theta) = \alpha_{\rho c} + \beta_{\rho c}.(\Theta + 273.15)$$

where $\beta_{\rho c}$ is a constant mean thermal coefficient and $\alpha_{\rho c}$ the heat capacity at temperature 0 K.

In order to take into account the heat of crystallization during the cooling of a magmatic body, we have used (Delaney, 1987):

$$\rho c(\Theta) = \rho c(\Theta) + L_f/(\Theta_{\text{liquidus}} - \Theta_{\text{solidus}})$$

which is valid only in the interval $[\Theta_{\text{solidus}}, \Theta_{\text{liquidus}}]$.

These three temperature-dependent relationships add a nonlinear loop to the solution process at each time step and the control criterion is the convergence rate of the temperature solution.

The time integration is a semi-implicit finite-difference scheme and a coefficient that can range between 0 and 1 determines the balance between the fully explicit and fully implicit finite-difference scheme. We have determined that 0.667 is a good compromise between precision of the solution and computing time. This coefficient can be adjusted as an input parameter.

3. Computer program

The initial version of THERMIC software was written in Borland Turbo Pascal® for MS-DOS® and it has not been fully rewritten in C, but translated from Turbo Pascal®. The C code resembles Pascal and the first version had to handle the 640 Kb memory limitation of all programs running under MS-DOS®. Various schemes used to bypass this memory limitation still remain in the present C version. Nevertheless, these coding issues may be avoided by referring to the comments contained in the source code.

The THERMIC package consists of four different programs: MESHGEN, THERMIC, OUT2XYT, THERMPRF.

3.1. The mesh generator, MESHGEN

Any structure, even one with holes, can be represented by a mesh whose elements are defined by polygons with 3 to 9 vertices, called nodes. Once a structure has been meshed, the solution of any continuous problem can be computed at each node.

MESHGEN creates a file containing the topology of the mesh, that is the links between nodes and their coordinates. This file is then used as input to the main program, THERMIC.

MESHGEN uses a macro-block (*.blc) file as input. This file contains all information necessary to generate a mesh for a structure described by macro-blocks (8-node blocks of the same material having homogeneous properties). The macro-blocks as well as the way to divide them into smaller elements are defined. Finally, the program builds 3, 4, 6, 8 or 9 node isoparametric elements (Zienkiewicz and Phillips, 1971).

The maximum number of elements and nodes is fixed using *#define* pre-compilation instructions contained in the *declarat.h* header file. They respectively default to 900 (*melem* variable) and 1350 (*mpoint* variable). These values may be changed by simply editing the *declarat.h* file and recompiling the whole source code. There is no firm upper limit to these constants despite the hardware memory limitations.

MESHGEN is also able to optimize the computed mesh by re-numbering the nodes to minimize the frontal width of the linear system to solve. This optimization speeds up the solution but leads to unnatural node numbering and thus creates some difficulties in entering boundary conditions.

A complete description as well as an example of a macro-block file is given in the *users guide* (downloadable from the web [1]).

3.2. The solver, THERMIC

The computing scheme of THERMIC can be described in four steps (Fig. 1):

1. reading the mesh data (*.mes file);
2. reading conditions of the thermal problem to solve (*.res file);
3. skyline matrices assembly and storage;
4. solution and output.

The solution scheme itself depends on the kind of problem being solved: steady or transient state, linear or nonlinear system and symmetric or non-symmetric geometry.

As already mentioned, one of the most useful features of THERMIC is its capability to handle nonlinear systems. The solution algorithm for such non linear systems is the following:

1. linearize the system;
2. solve the associated linear system;
3. compute the norm to test the convergence;
4. repeat steps 1, 2, 3 until convergence is achieved (norm lower than a given variable called *seuil*) or until the maximum number of iterations is reached (*nbremaxinl* variable).

Both variables *seuil* (default 0.03) and *nbremaxinl* (default 20) are defined and may be changed by using *#define* directives within the header file *declarat.h*.

3.3. Post-processing of results, THERMPRF and OUT2XYT

3.3.1. THERMPRF

THERMPRF extracts temperature vs. time or vs.

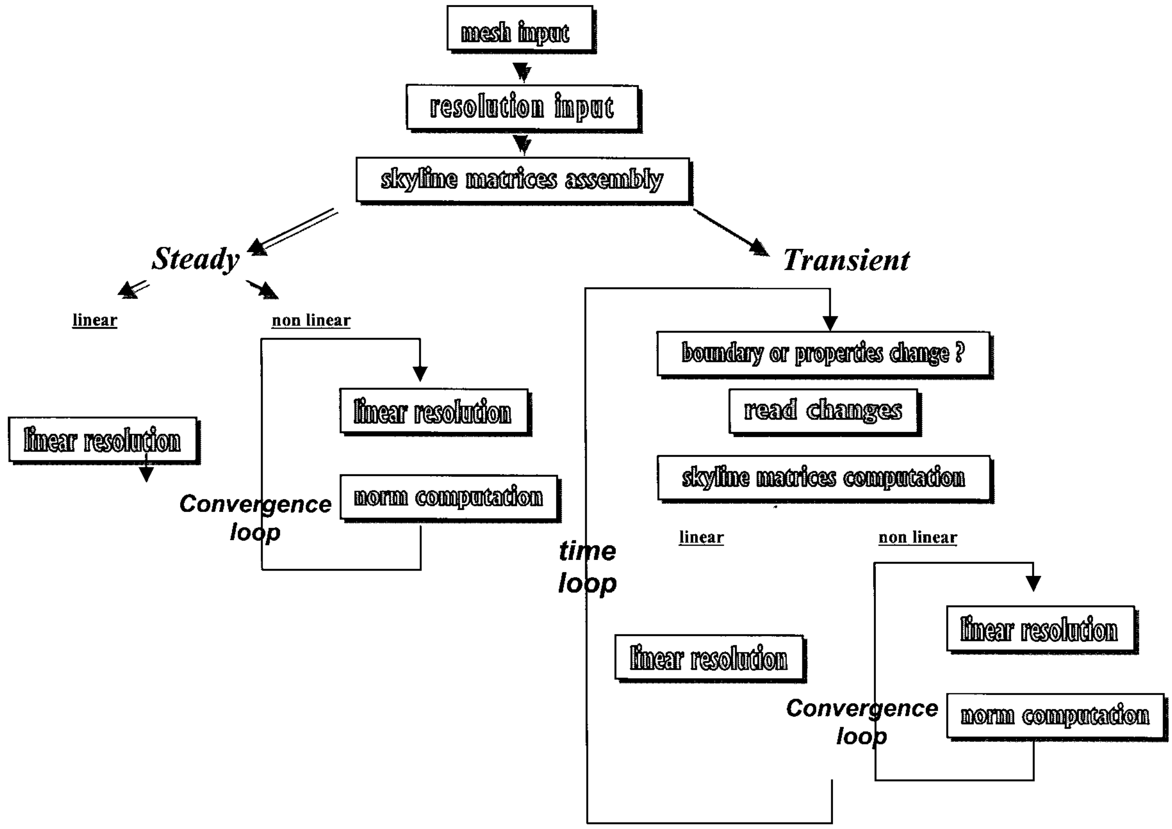

Fig. 1. Outline of problem solution scheme.

distance profiles from any output file of a THERMIC run.

3.3.2. OUT2XYT

OUT2XYT converts any output file from THERMIC into an x, y, t file where x and y are the coordinates of the nodal points and t the temperature. This xyt output file may be easily entered into an end-user-preferred plotting tool to obtain isotherm plots like the one shown on Fig. 2 (e.g. GMT software by Wessel and Smith, 1991).

3.3.3. ISOTHERM

ISOTHERM is a supplementary program that creates 2D contour plots of temperature from any THERMIC's output file (Fig. 2). It uses the same nodal functions and finite-element grid as the one used in the solution of the problem. ISOTHERM is only available on PC as an executable file (consult our WEB site for the future release of the source code and UNIX versions).

4. General computing flow

The whole process of using THERMIC is represented on Fig. 3. A limited number of questions are posed to the user within a THERMIC session, most of which have default answers. The main information concerning the problem to solve is provided using input files. Thus the most difficult steps consist of writing both the macro-block (*.blc) and the solution (*.res) files. The *user's guide* gives the complete instructions for building these files.

The macro-block file (*.blc), describing the structure to model as a set of super-elements, is used by MESHGEN to generate a mesh to be stored in a *.mes file. An option allows the nodes of the final mesh to be re-numbered in order to optimize the solution scheme.

The solution file (*.res) applied to a specified mesh (*.mes file) describes the problem to be solved. It contains the following information:

- type of state (steady or transient);
- type of solution: linear or non linear (whether the properties of the material depend on the temperature or not);
- number and properties of the materials;

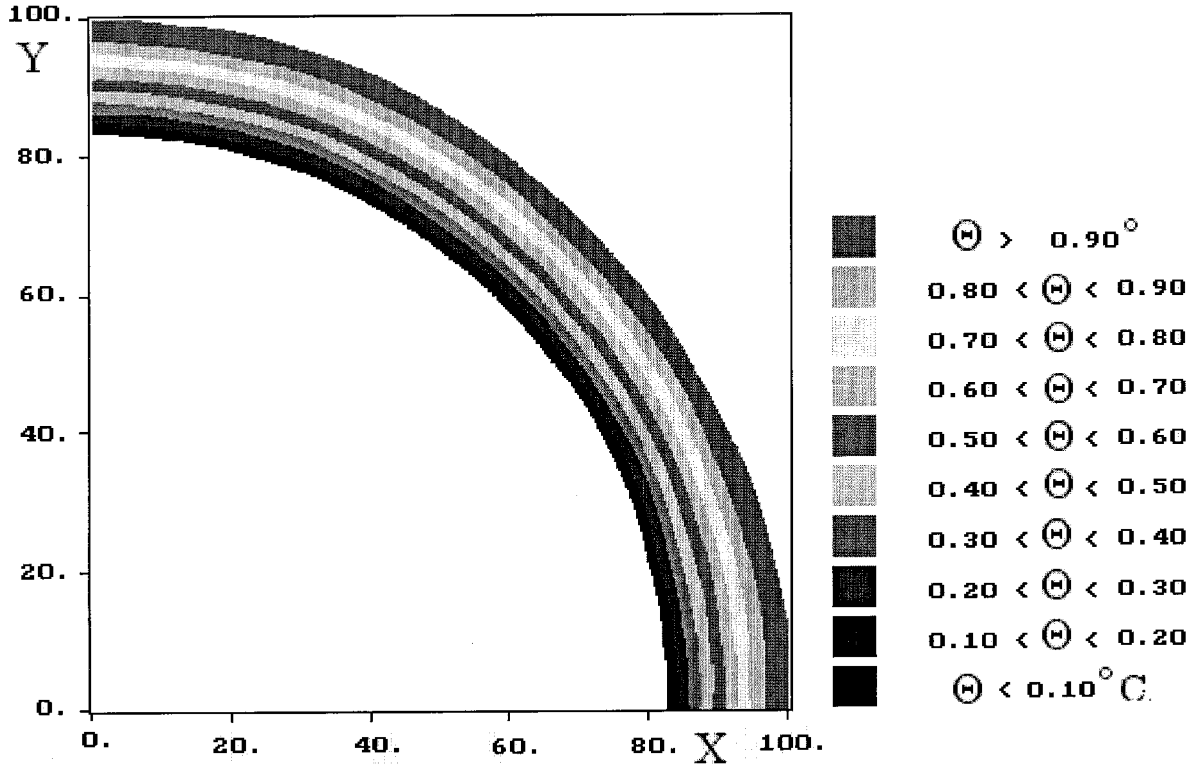

Fig. 2. Example of isotherm plot for sphere with prescribed temperatures on boundaries (steady-state case, homogeneous properties and axisymmetry).

- mixed type conditions, prescribed temperatures, advective term, changes of problem conditions;
- initial conditions (either given explicitly in the solution file or derived from a previous THERMIC run);
- time step, unit of time, duration of modelling.

Two output files will be generated:

1. a *.lst file that contains formatted results with all the parameters of the problem and the mesh data (*optional*);
2. a *.out file that contains unformatted results for plotting purposes or to be used as initial conditions for a further run of THERMIC on the same mesh.

Two environment files are required in the current directory (default) or in the directory specified during the compilation process:

- one file containing the user's messages. The filename extension defines the language: messther.ENG and messther.FRE for English and French messages respectively;
- Thermic.ini contains the code of the language for the THERMIC program messages (ENG for English, FRE for French).

Examples of all input and output files used by THERMIC are given in the appendices.

5. Testing and examples

THERMIC has been tested on various problems with known analytical solutions (Carslaw and Jaeger, 1986) involving all allowed boundary conditions, heat production and for both transient and steady-state cases. Results of these tests are available on request from the authors.

Some applications already exist in volcanism (cooling of a magma chamber; Chery et al., 1991), paleomagnetism (cooling of dykes and lava flows; Smith et al., 1991), metamorphism (study of a granitic magma ascent; Moinet et al., 1989) and fluid flows in a sedimentary basin (Vasseur et al., 1993).

We have chosen to illustrate this presentation with the example of a cooling dyke (Smith et al., 1991). In that study, the authors compared a model of cooling dyke to a profile of paleotemperatures observed in a Permian red sandstone layer intruded by a Plio–Pleistocene basaltic dyke.

During dyke injection, the flowing magma advects

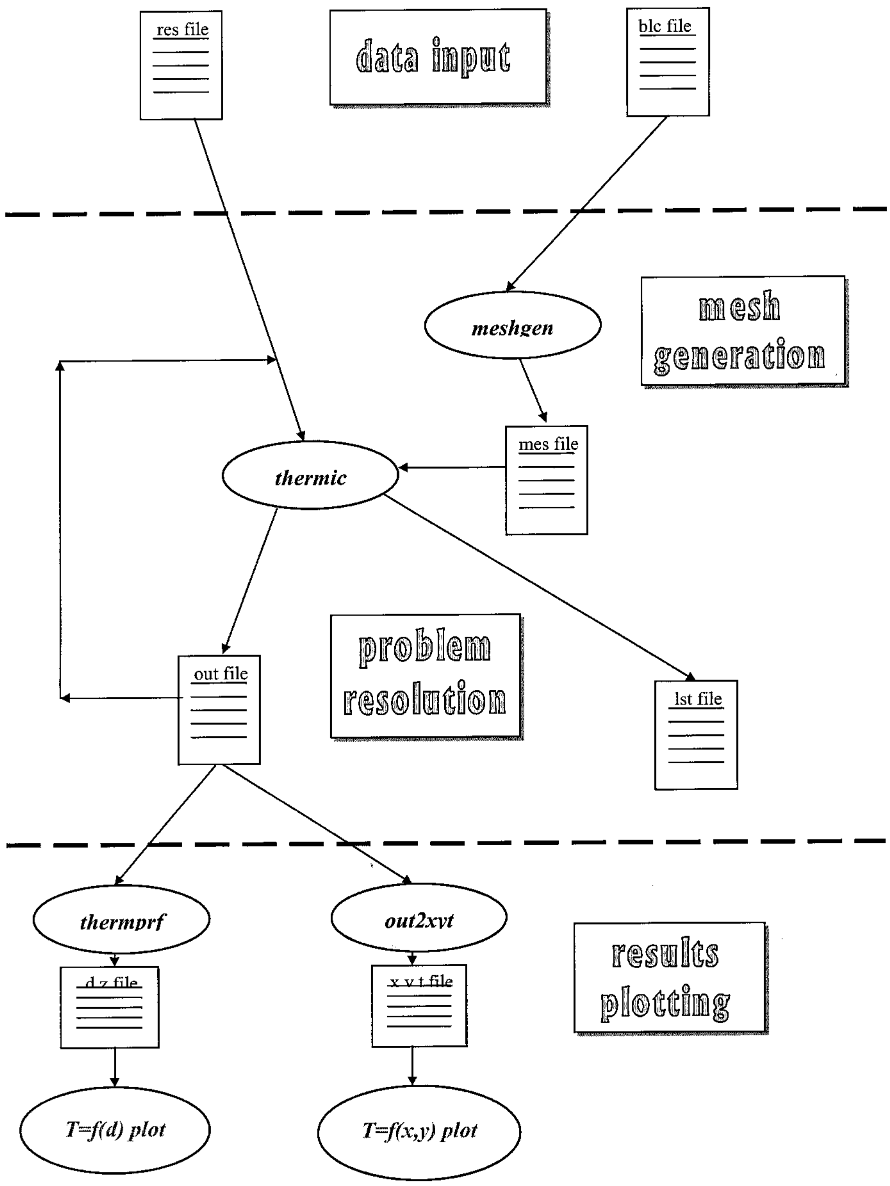

Fig. 3. Simplified flow chart showing input and output of THERMIC.

heat through the dyke, while heat is conducted out of the dyke into the surrounding rock. Several analytical models exist for simple geometry to calculate the temperature in and around the magmatic intrusion, but none of them can take into account simultaneously the effects of the following parameters that could play an important role in such phenomena:

- thermal diffusivity, $\kappa = K/\rho c$, of both the dyke and the host rock and its temperature dependence which enters through the temperature dependence of the thermal conductivity $K = \alpha_K + \beta_K/(\Theta + 273)$ established from empirical considerations (Hanley et al., 1978); the product of the density ρ and the heat capacity of the rock, c, is assumed to be constant;
- latent heat of crystallization of the basalt, L_f;
- flow duration of the magma;
- width of the flowing magma in the central part of the dyke relative to the thickness of the possibly solidified basaltic layer at the channel periphery.

Our physical model based on a two-dimensional box, 100 m wide and 200 m high, is described on Fig. 4. The abscissa x = 0 corresponds to the dyke axis and the reference level is placed at y = 100 m which corresponds to the probable maximum depth of burial for the sand-

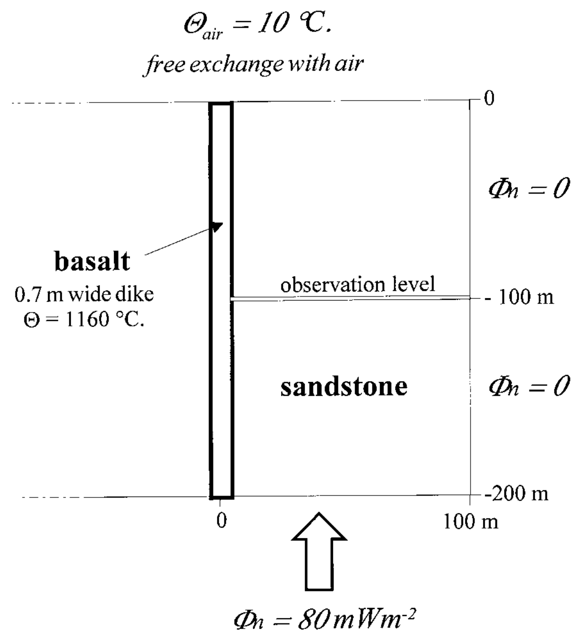

Fig. 4. Physical model of basaltic dyke cooling with boundary conditions.

stone bed at the time of intrusion. The medium is divided into 36 elements (Fig. 5): 8 belong to the basalt and 28 to the sandstone. The mesh is dense and the elements regularly spaced along x in the basalt and in the sandstone up to x = 1.64 m; beyond that, the 2 remaining elements are much wider. Each element contains 9 nodes, so that the temperatures are calculated over a total of 185 nodes, every 4.38 cm in the basalt and every 5 cm in the sandstone up to 1.64 m.

5.1. Boundary conditions and initial conditions

Before the dyke injection, the horizontal heat flux is zero through the two vertical planes of the box ($x = 0$ and $x = 100$ m). At the bottom, we assume a vertical flux of 80 mW m^{-2} which corresponds to a normal geothermal gradient in the absence of heat production. At the top, we assume a free exchange between the earth's surface and the air which gives a surface temperature fluctuating around 10°C. Calculation of the temperature distribution in this static problem places the 12°C isotherm at the reference level.

At $t = 0$, the dyke is instantaneously intruded and a temperature of 1160°C is prescribed at all the nodes associated with the dyke, while a temperature of 586°C (intermediate between 1160°C and the host rock temperature) is assumed for the nodes at the basalt/sandstone interface.

The transient problem which results from the dyke injection has been calculated for several cases:

Case 1: The dyke is injected instantaneously and subsequently cools laterally by conduction into the sandstone;

Case 2: Subsequent to its injection, the magma is allowed to flow for a prescribed time with uniform and constant temperature;

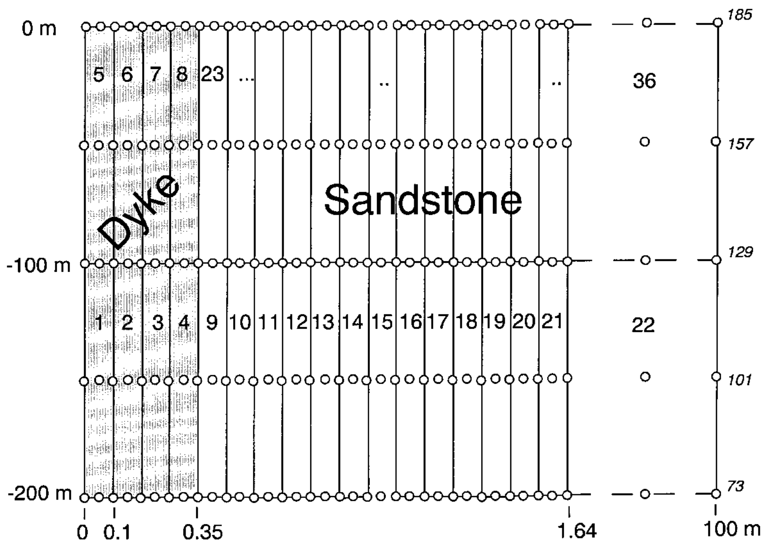

Fig. 5. Mesh used to solve physical problem shown in Fig. 4. Structure is composed of 36 9-node elements corresponding to 185 nodes (numbers in italics). Grey area to the left corresponds to dyke.

Case 3 and 4: The magma is allowed to flow with uniform and constant temperature through a certain width around its median plane.

Each of the previous cases is subdivided into 3 subcases:

(a) the thermal conductivity is temperature-independent and there is no latent heat of crystallization ($L_f = 0$).
(b) the thermal conductivity is temperature dependent ($K = f(\theta)$) in the basalt and in the sandstone and the latent heat of the basalt is included ($L_f > 0$).
(c) the effects of natural convection of pore water in the host rock is approximated by increasing the lattice conductivity in the sandstone by a factor Nu, the Nusselt number, between values of 1 and 4, $K = Nu\,K_0$. The latent heat of the basalt is also considered.

A simple model involving instantaneous emplacement immediately followed by cooling (case 1) is unrealistic for the dyke simulation (Smith et al., 1991). The best result is obtained for case 4, subcase c, with a 44 cm wide flowing channel in the dyke, temperature dependence for K and $L_f > 0$ and with limited natural convection in the sandstone (the curves corresponding to the subcases are represented in Fig. 6).

6. Computational performance

Table 1 summarises a few benchmarks of the program applied to two different meshes and running on three computing platforms:

- Sun SparcStation 5®, 64 Mb RAM, running Solaris 2.5®.
- PC Pentium II 300 MHz, 256 Mb RAM, running Windows NT 4.0® Workstation.
- PC Pentium MMX 200 MHz, 16 Mb RAM, running Windows 95®.

THERMIC was compiled with the MS Visual C++® compiler on the PC's platforms and the Gnu C compiler on the SparcStation.

6.1. Availability and hardware requirements

THERMIC is available in two versions: Unix or MS Windows (95, 98, NT) system-based computers. The two packages are available on the web with a complete installation procedure (*THERMIC installation guide*) and a users guide (*THERMIC user's guide*). The THERMIC package has been successfully tested on SUN® and SGI® UNIX workstations and on Microsoft Windows 95® and Windows NT® 4.0 system based PCs. The package contains source files (*.c, *.h), makefiles (Makefile, *.mak) and environment files

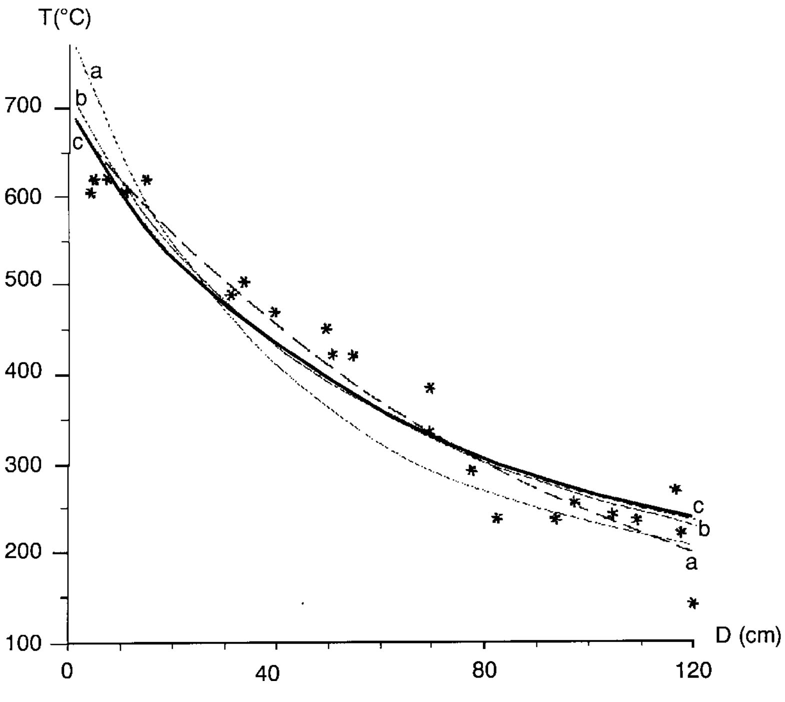

Fig. 6. Temperature vs. distance in sandstone for magma flowing through 44 cm wide central channel in dyke. Dashed line corresponds to best-fitted curve to observed data (stars) and three dotted lines a, b and c to subcase models (see text). Best model (subcase c) is shown as thicker line.

Table 1
Running time of THERMIC in 7 different cases (4, 20, 400 and 900 9-node-elements) with following boundary conditions: (1) and (2) 3 prescribed temperatures; (3) and (4) 6 prescribed temperatures; (5) 42 prescribed temperatures; (6) 82 prescribed temperatures; (7) 122 prescribed temperatures

Case	Mesh (elements)	Solution	State	Advection	Running time (s)		
					Sun SS 5 Solaris 2.5	PC P II 300 NT 4.0 W	PC P 200 Win 95
1	4	linear	steady	yes	0.16	0.30	0.55
2	4	linear	transient (20 steps)	yes	0.20	0.38	0.65
3	4	nonlinear	steady	no	0.13	0.30	0.50
4	4	nonlinear	transient (10 steps)	no	0.20	0.35	0.65
5	20	linear	steady	no	0.23	0.40	0.87
6	400	linear	steady	no	3.80	1.60	12.75
7	900	linear	steady	no	13.10	4.05	24.25

as well as executable files for both systems. The user may directly run the executables or recompile the whole program using any standard Unix C compiler or the MS Visual C++® compiler. The package also includes a html directory with help and example files.

6.1.1. THERMIC for UNIX

THERMIC has been successfully compiled on Sun Sparc Stations® running Solaris® and on Silicon Graphics International Stations running Irix® using either the SunSparcs® compiler, standard compiler delivered with the machine or the Gnu C compiler (gcc).

The package is a compressed tar file thermic_unix.tar.Z (or thermic_unix.tar.gz). Uncompress (use gzip or *uncompress* depending on the archive file available) and unpack the archive file (use *tar* command).

6.1.2. THERMIC for MS Windows

THERMIC has been successfully compiled on different PCs running Windows95®, 98® and Windows NT 4.0® using the MS Visual C++® compiler. The package is a zip file, thermic_pc.zip, compressed with winzip.

7. Conclusion

Tested on several platforms and already applied with success to several geological problems, THERMIC provides an easy and rapid solution to most of the thermal problems that a geoscientist may encounter in research or teaching. Due to its modularity, THERMIC can be easily upgraded and adapted to specific problems with more detailed boundary conditions.

Acknowledgements

This program was written between 1987 and 1990 by Alain Bonneville while he was associated with the Centre Géologique et Géophysique, Université de Montpellier II, Centre National de la Recherche Scientifique (CNRS). We gratefully thank Dr Guy Vasseur for his help and advice during all these years and Dr Francis Lucazeau for his major contribution to the meshing program MESHGEN and many fruitful discussions. Translation from Turbo Pascal to C language was partly performed by Frederick Camuzard during his 6 weeks stay in Papeete as part of his undergraduate studies in computer sciences. We also wish to thank Dr R.Von Herzen for his comments and Dr P. Bird and R. Ketcham for their constructive reviews.

Appendix A. Example of mesh file (Fig. 5)

```
DYKE VERTICAL
 22 115   9   2
  1   1   2   3  12  21  20  19  10  11   1
  2   3   4   5  14  23  22  21  12  13   1
  3   5   6   7  16  25  24  23  14  15   1
  4   7   8   9  18  27  26  25  16  17   1
  5  19  20  21  30  39  38  37  28  29   1
  6  21  22  23  32  41  40  39  30  31   1
  7  23  24  25  34  43  42  41  32  33   1
  8  25  26  27  36  45  44  43  34  35   1
....................
  1     0.00000     0.00000
  2     0.12500     0.00000
  3     0.25000     0.00000
  4     0.37500     0.00000
  5     0.50000     0.00000
  6     0.62500     0.00000
  7     0.75000     0.00000
  8     0.87500     0.00000
..................
```

Appendix B. Example of solution file (in input)

```
Cooling of a dyke
trans
lin
0 3 3
2 0.0 1200
2 4
0.0 2.6 2.6 3.0E+6 0.0 522.0 522.0 9.0e+8 950.0
0.0 2.6 2.6 3.0E+6 0.0 522.0 522.0 0.0e+8 1950.0
0
90
6 300
7 300
8 300
9 300
....
0
20 0.667 2 1 jrs 86400
0
no
1   1150.0
5   1150.0
10   1150.0
14   1150.0
19   1150.0
23   1150.0
28   1150.0
32   1150.0
37   1150.0
41   1150.0
0
```

Appendix C. Example of output file

```
Finite Elements THERMIC - Version3.00 |
Universite de Polynesie francaise - Tahiti - French Polynesia 1999
Real Precision - Number of bytes =  8

THERMIC Program : Thermal Problems solver.
 Cooling of a dyke

Regime type       : transient
Mesh filename     : ../thermic/test/src/mes/dyke.mes
Input filename    : ../thermic/test/src/res/dyke2_eng.res
Listing filename  : ../thermic/test/resultats/dyke2_eng.lst
Output filename   : ../thermic/test/resultats/dyke2_eng.out
All temperature in Celsius Degrees.

Mesh Title : DYKE VERTICAL

Number of Elements        : 22
Number of Nodes           : 115
Number of Node per Element : 9
Number of dimensions      : 2
System solving type :linear.
Prescribed Temperature Scale  :
Min.Temp.=  0.00°C. -  Max.Temp.= 1200.00°C.
Number of materials           :  2
Number of properties per material :  4
Materiau #   Heat Prod.    Kx       Ky      Rhoc     Rhocf
            1         0       2.6      2.6     3e+06
     2          0       2.6       2.6     3e+06
Number of boundary convection conditions :   0
Number of prescribed temperatures :  90
   6= 300.00  7= 300.00  8= 300.00  9= 300.00  15= 300.00  16= 300.00  17= 300.00  18= 300.00
  24= 300.00  25= 300.00  26= 300.00  27= 300.00  33= 300.00  34= 300.00  35= 300.00  36= 300.00
  42= 300.00  43= 300.00  44= 300.00  45= 300.00  46= 300.00  47= 300.00  48= 300.00  49= 300.00
  50= 300.00  51= 300.00  52= 300.00  53= 300.00  54= 300.00  55= 300.00  56= 300.00  57= 300.00
  58= 300.00  59= 300.00  60= 300.00  61= 300.00  62= 300.00  63= 300.00  64= 300.00  65= 300.00
  66= 300.00  67= 300.00  68= 300.00  69= 300.00  70= 300.00  71= 300.00  72= 300.00  73= 300.00
  74= 300.00  75= 300.00  76= 300.00  77= 300.00  78= 300.00  79= 300.00  80= 300.00  81= 300.00
  82= 300.00  83= 300.00  84= 300.00  85= 300.00  86= 300.00  87= 300.00  88= 300.00  89= 300.00
  90= 300.00  91= 300.00  92= 300.00  93= 300.00  94= 300.00  95= 300.00  96= 300.00  97= 300.00
  98= 300.00  99= 300.00 100= 300.00 101= 300.00 102= 300.00 103= 300.00 104= 300.00 105= 300.00
 106= 300.00 107= 300.00 108= 300.00 109= 300.00 110= 300.00 111= 300.00 112= 300.00 113= 300.00
 114= 300.00 115= 300.00

Time steps number          :  20
Time scheme coefficient     : 0.667
Time step duration          : 2.000000E+00 jrs

Initial conditions are taken into account :
   1 1150.00 |  2 1150.00 |  3 1150.00 |  4 1150.00 |  5 1150.00 |  6  300.00 |
   7  300.00 |  8  300.00 |  9  300.00 | 10 1150.00 | 11 1150.00 | 12 1150.00 |
  13 1150.00 | 14 1150.00 | 15  300.00 | 16  300.00 | 17  300.00 | 18  300.00 |
  19 1150.00 | 20 1150.00 | 21 1150.00 | 22 1150.00 | 23 1150.00 | 24  300.00 |
.................

Total number of equations  :  25
Symmetrical solving matrix.
average band height        :  7- maximum :  12
Column height              :
  0  1  2  1  2  5  6  7  6  7  10  11  12  11  12  5  6  7  6  7  10  11  12  11  12
Diagonal terms position     :
  1  3  6  8  11  17  24  32  39  47  58  70  83  95  108  114  121  129  136  144  155  167  180  192  205
Off-diagonal terms number   :  180
Total number of terms in the matrix :  205
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 Results
---------------------------------------------------
   Step  5 - Time  10.000000 jrs
 1  310.389555  |  2  310.205917  |  3  309.432670  |  4  307.154090  |  5  296.592217  |  6  300.000000* |
 7  300.000000* |  8  300.000000* |  9  300.000000* | 10  310.389555  | 11  310.205917  | 12  309.432670  |
13  307.154090  | 14  296.592217  | 15  300.000000* | 16  300.000000* | 17  300.000000* | 18  300.000000* |
19  310.389555  | 20  310.205917  | 21  309.432670  | 22  307.154090  | 23  296.592217  | 24  300.000000* |
25  300.000000* | 26  300.000000* | 27  300.000000* | 28  310.389555  | 29  310.205917  | 30  309.432670  |
31  307.154090  | 32  296.592217  | 33  300.000000* | 34  300.000000* | 35  300.000000* | 36  300.000000* |
37  310.389555  | 38  310.205917  | 39  309.432670  | 40  307.154090  | 41  296.592217  | 42  300.000000* |
43  300.000000* | 44  300.000000* | 45  300.000000* | 46  300.000000* | 47  300.000000* | 48  300.000000* |

49  300.000000* | 50  300.000000* | 51  300.000000* | 52  300.000000* | 53  300.000000* | 54  300.000000* |
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